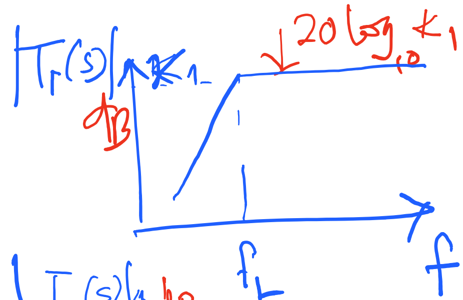
Bode Plot

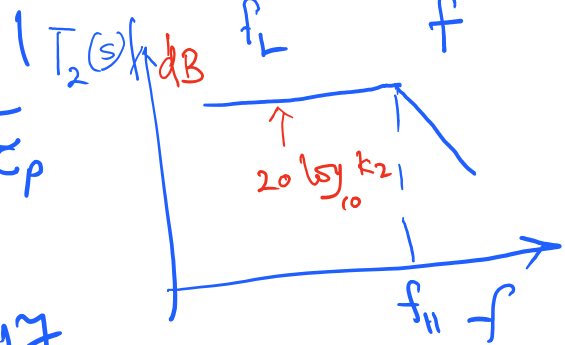
Monday, September 19, 2016 8:24 AM

$$T_1(s) = K_1 \frac{s\tau_s}{1+s\tau_s}$$



$$\boxed{\begin{aligned} \angle\theta_2 &= 90 \\ A_2 &= \omega\tau_s \end{aligned}}$$

$$T_2(s) = K_2 \frac{1}{1+s\tau_p}$$



$A_2\angle\theta_2 = s\tau_s = j\omega\tau_s$

$A_1\angle\theta_1 = K_1$

$A_1 = K_1$

$\theta_1 = 0$

$$= A\angle\theta = x + jy$$

$$= Ae^{j\theta} = A_1 e^{j\theta_1} A_2 e^{j\theta_2} \cdots$$

Bode Plot

frequency response → dB

1) Magnitude vs freq

2) Phase vs freq

dB

Decibel

$$\text{Bel} = \log_{10}\frac{P_o}{P_{in}}$$

$$\text{dB} = 10\log_{10}\frac{P_{out}}{P_{in}} = 10\log_{10}\frac{v_o^2/\cancel{R_o}}{v_i^2/\cancel{R_i}}$$

$$= 10\log_{10}\left(\frac{v_o}{v_i}\right)^2$$

$$= 20\log_{10}\frac{v_o}{v_i} = 20\log_{10}\frac{i_o}{i_i}$$

$$\ln T(s) = \ln\left(A_1 e^{j\theta_1} A_2 e^{j\theta_2} \cdots\right) \cdots$$

$$\ln T(s) = \ln\left(A_1 e \;\; A_2 \cdots\right)$$

$$= \ln\left[A_1 A_2 \cdots e^{j(\theta_1+\theta_2+\cdots)}\right]$$

$$= \ln(A_1 A_2 \cdots) + \ln\left[e^{j(\theta_1+\theta_2+\cdots)}\right]$$

$$= \ln A_1 + \ln A_2 + \cdots + j\left(\theta_1+\theta_2+\cdots\right)$$

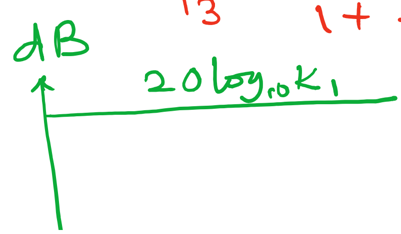
$$= 1 + s\tau_s = A_3 \angle\theta_3 = 1 + j\omega\tau_s$$

$$\boxed{A_3 = \sqrt{1^2 + (\omega\tau)^2}} \qquad \boxed{\theta_3 = \tan^{-1}\frac{\omega\tau_s}{1}}$$

$$\left[\begin{array}{l} T(s)\big|_{dB} = 20\log_{10} A_1 + 20\log_{10} A_2 + \cdot 20\log_{10} A_3 \\ \angle T(s) = \theta_1 + \theta_2 + \theta_3 \; - \end{array}\right.$$

$$T(s) = T_1 T_2 T_3$$

$$T_1 = K_1$$
$$T_2 = s\tau_s$$
$$T_3 = \frac{1}{1+s\tau_s}$$

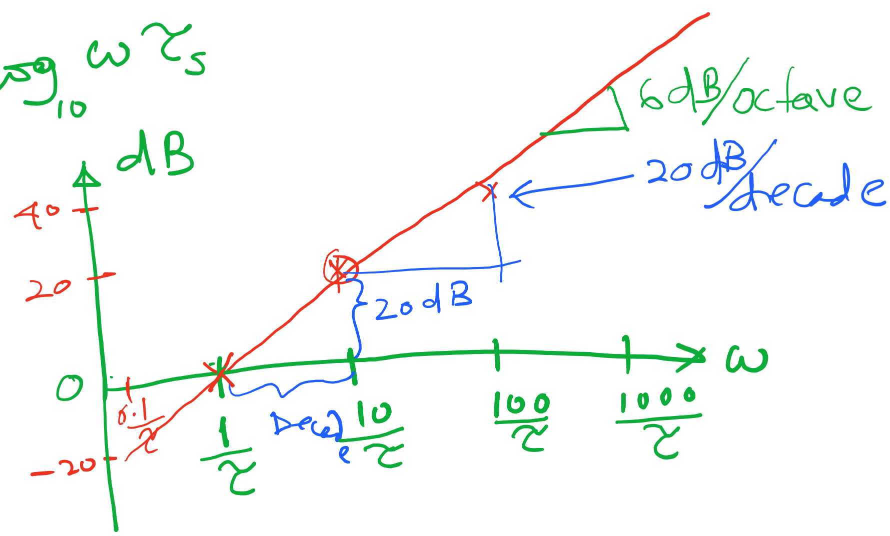


$$T_1(s)\big|_{dB} = 20\log_{10} K_1$$

$$T_1(s)\big|_{dB} = 20\log_{10} K$$

$$T_2(s) = T_2 = s\tau_s = j\omega\tau_s$$
$$= \omega\tau_s \angle 90^\circ$$

$$T_2\big|_{dB} = 20\log_{10}\omega\tau_s$$



$\omega = \frac{0.1}{\tau_s}$

$\omega = \frac{1}{\tau_s}$

$\downarrow$

$20\log_{10} 1 = 0$

$\omega = \frac{10}{\tau_s}$

$20\log_{10} 10 = 20\,dB$

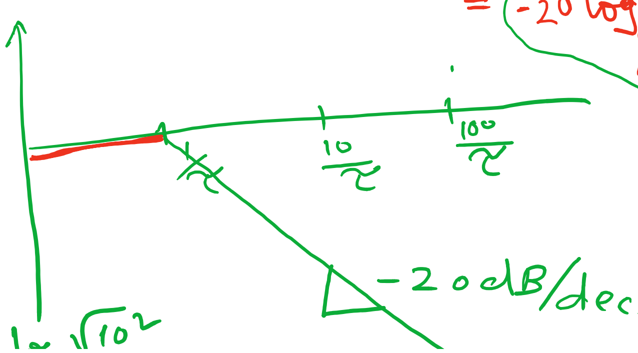
$$T_3(s) = \frac{1}{1+s\tau_s} = \frac{1}{1+j\omega\tau_s} = \frac{1}{\sqrt{1+(\omega\tau)^2}\angle\tan^{-1}\omega\tau}$$

$$T_3\big|_{dB} = 20\log_{10}\frac{1}{\sqrt{1+(\omega\tau)^2}} = -20\log_{10}\sqrt{1+(\omega\tau)^2}$$

$= 0 \quad \omega < \frac{1}{\tau}$

$= -20\log_{10}\omega\tau \quad \omega > \frac{1}{\tau}$

$\omega = \frac{1}{\tau}$



$\omega = \frac{10}{\tau}$

$-20\log\sqrt{1+10^2}$

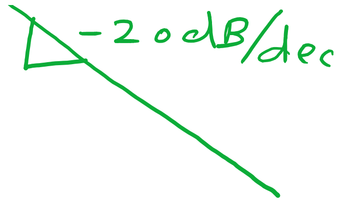
$\approx \sqrt{10^2}$

$-20\log_{10}\sqrt{1+1^2}$

$= -20\log_{10}\sqrt{2}$

$= -3\,dB$

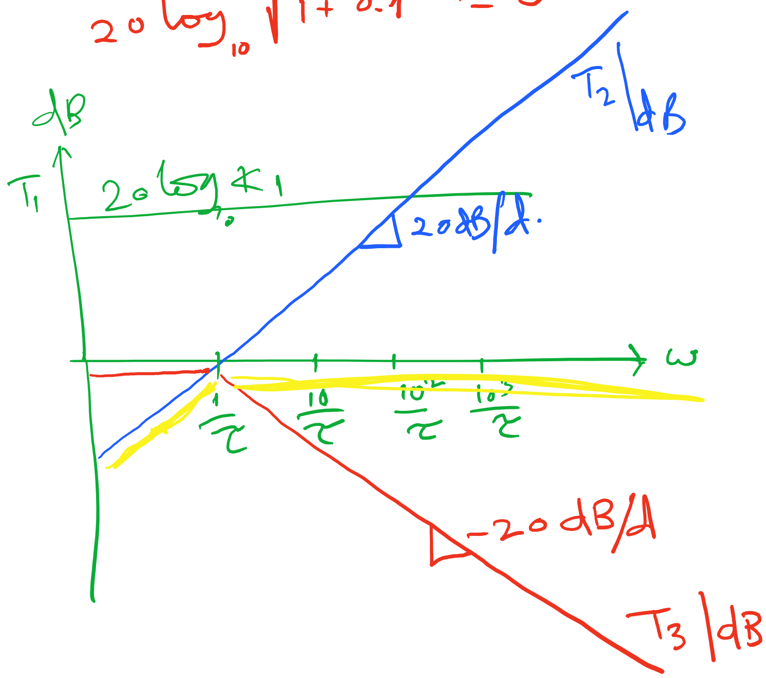
$$-20\log_{10}\sqrt{1+10^2}\;\Big|\; = -20\,dB = -20\log_{10}\sqrt{10^2}$$

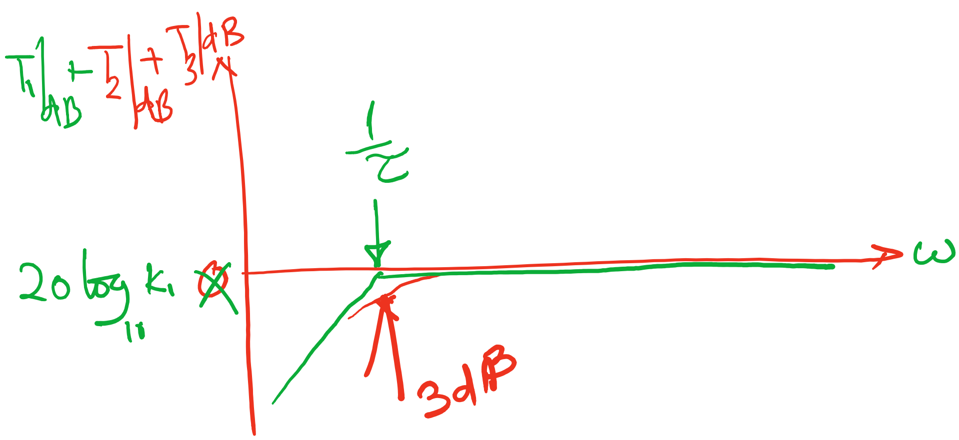


$$= -20\log_{10}\sqrt{2}$$
$$= -3\,dB$$
$$\approx 0$$

$$\omega = \frac{0.1}{\tau}$$

$$20\log_{10}\sqrt{1+0.1^2} \simeq 0$$





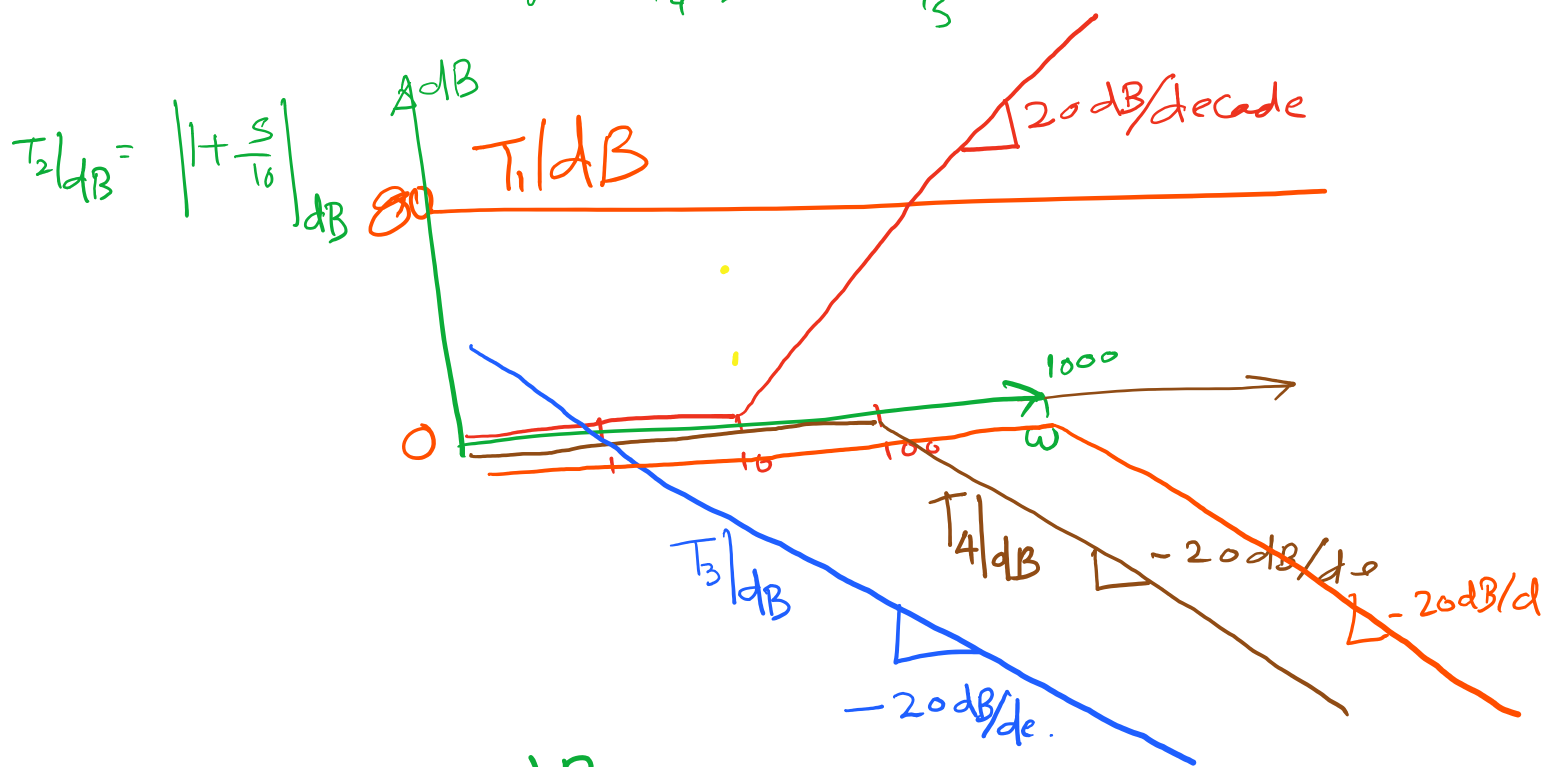
$$\# \quad T(s) = \frac{10^8(10+s)}{s(100+s)(1000+s)}$$

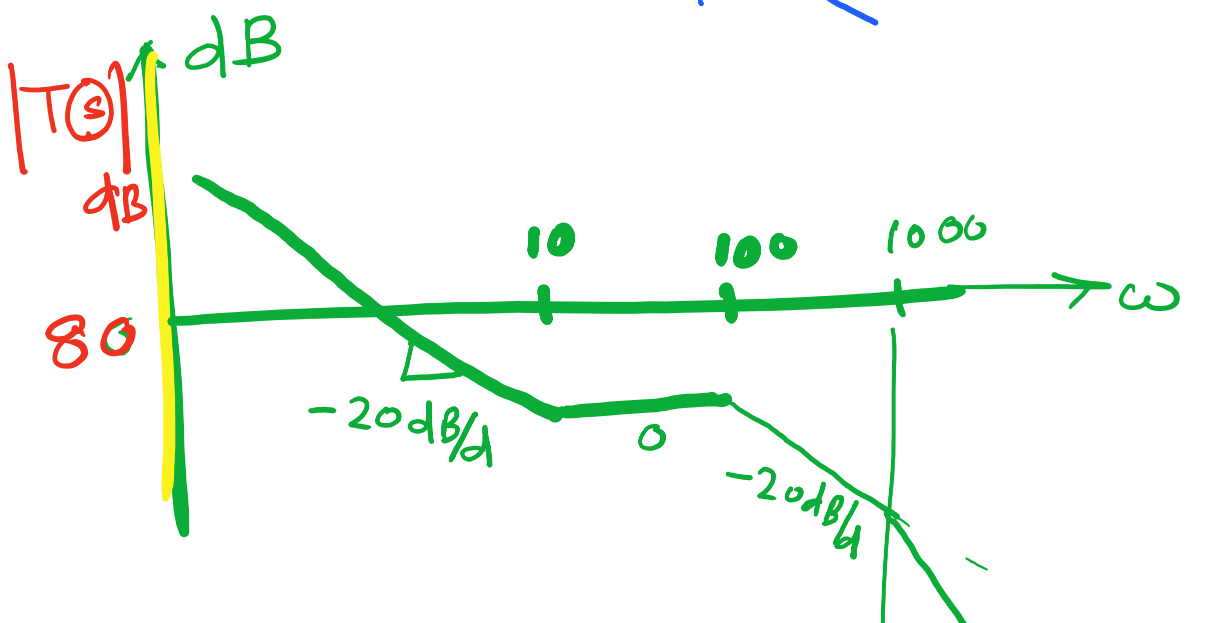
$$= \frac{10^{\cancel{8}\,3}\;10\left[1+\frac{s}{10}\right]}{s\;\cancel{100}\left[1+\frac{s}{100}\right]\cancel{1000}\left[1+\frac{s}{1000}\right]}$$

$$= \frac{\overset{T_1}{10^4}\ \overset{T_2}{\left[1+\frac{s}{10}\right]}}{\underset{T_3}{s}\ \underset{T_4}{\left[1+\frac{s}{100}\right]}\ \underset{T_5}{\left[1+\frac{s}{1000}\right]}}$$

$\omega_1 = 10$

$\omega_0 = 1$ $\quad \omega_2 = 100$ $\quad \omega_3 = 1000$





-40dB/d

$$20\log_{10}\frac{A_{vmax}}{\sqrt{2}} = 20\log_{10}A_{vmax} + 20\log_{10}\frac{1}{\sqrt{2}}$$

$$\boxed{= 20\log_{10} A_{vmax} - 3\,dB} + 20\log_{10}\frac{1}{\sqrt{2}}$$